Changes in an option's fair value

You can use this exercise and the option pricer on the website to enhance your understanding of option valuation principles. You may find it easier to print off this exercise.

The pricer can be found on

http://www.fmarketstraining.com/option-pricer.htm

It's important to remember that the price of an option is determined by demand and supply – all an option model will tell you is how the option will change in value as a result of a change in market parameters.

Exercise

In this exercise we will investigate how the value of a call option struck at the money forward will change in price as the underlying market factors change.

At each stage we will change one factor while holding all of the other factors constant.

Set the option pricer with the following parameters:

Type:	Call
Spot:	$15
Strike:	$15.15
Maturity:	1 (year)
Funding:	3%
Yield:	2%
Volatility:	30%

The BSM model returns an initial fair value of $1.7534 for both the call and the put

Consider now how the value of the two options changes as the individual underlying market parameters change but all of the other factors are held constant.

Change in the spot price

Complete the following box by changing only the spot price.

Spot price	Value of a call ($)	Change in value of call	Value of a put ($)	Change in value of put
$13	0.84		2.80	
$14	1.25	0.41	2.23	-0.57
$15				
$16				
$17				
$18				

Table 1 Impact on the value of a call and a put from a change in the spot price

Learning points

Option theory decomposes the premium into two components: intrinsic and time value. Intrinsic value is the amount that the holder of the option would realise if he could exercise his option straight away. If the option is European in style this would not be possible so we could redefine this as the advantage that the option confers to holder over the underlying price. However, since the underlying is a forward contract it is defined as:

Intrinsic value for a call option = MAX (forward price – strike price, 0)

Intrinsic value for a put option = MAX (strike price – forward price, 0)

The time value component of an option premium can be thought of as the potential future intrinsic value and is primarily a function of implied volatility and time to maturity.

From table 1 we can highlight a number of features:

- For a single unit increase in the underlying price the value of the option increases by less than one unit.

- The relationship between the underlying price and the value of the option prior to expiry is non-linear.
- When the option is OTM and ATM the premium comprises entirely of time value.
- Time value is greatest when the option is ATM.
- When the option is ITM the premium consists of increasing intrinsic value and decreasing time value.

Change in time to maturity

Make sure you reset the spot price to $15.00. Complete the following box by changing only the time to expiry.

Time to expiry	Value of a call ($)	Change in value of call	Value of a put ($)	Change in value of put
1.00	1.75		1.75	
0.80	1.56	-0.19	1.59	-0.16
0.60				
0.40				
0.20				
0.00				

Table 2 Impact on the value of a call and put from the passage of time

Table 2 shows that the passage of time has a negative effect on both the call and the put option and that this accelerates as the option approaches expiry.

Learning points

An alternative approach to analysing table 2 is to compare the different option premiums with respect to their maturity. Intuitively one might expect a 12 month option to be worth twice as much as a six month option, however, this is not the case. A six month call option priced using the same parameters costs $1.22 while a 12 month option is valued at $1.75. The mathematics behind the calculation is:

$$
\begin{aligned}
\text{12 month variance} &= 2 \times \text{6 month variance} \\
\text{12 month standard deviation} &= \sqrt{2 \times \text{6 month variance}} \\
&= \sqrt{2} \times \sqrt{\text{6 month variance}} \\
&= \sqrt{2} \times \text{6 month standard deviation}
\end{aligned}
$$

This suggests that the approximate premium on a 12 month option should be:

$$
\begin{aligned}
&= \$1.22 \times \sqrt{2} \\
&= \$1.22 \times 1.41 \\
&= \$1.73
\end{aligned}
$$

As we saw before the premium calculated by the model was $1.75. From this it follows that buying two six month options will be more expensive than buying one longer dated option. Turning the argument around selling two consecutive options will generate more income than one longer dated option.

Change of implied volatility

Make sure you reset the maturity to 1 year. Complete the following box by changing only the time to expiry.

Implied volatility	Value of call ($)	Change in value of call	Value of put ($)	Change in value of put
0.00%	0.00		0.00	
10.00%	0.59	0.59	0.59	0.59
20.00%				
30.00%				
40.00%				
50.00%				

Table 2 Impact on the value of a call and a put from a change in implied volatility

Learning points

Table 2 illustrates that as implied volatility increases both options will increase in value. Each 10% rise in implied volatility will increase the premium on both options by a constant (ish) amount. Two other features are also worth mentioning. Note that an option that has no intrinsic value and is priced using 0% volatility is worthless. Zero volatility suggests that the underlying price is not expected to move over the remaining life of the option and so the options will expire worthless. The table also highlights that the relationship is proportional; doubling the volatility doubles the premium.

Table 3 looks at the impact of a change in implied volatility on an ITM and OTM call. Both options are valued using the original strike of $15.15 but the spot price is assumed to be $20 and $10 for the ITM and OTM positions respectively.

Implied volatility	**Value of ITM call ($)**	**Change in value of call**	**Value of OTM call ($)**	**Change in value of put**
0.00%	4.90		0.00	
10.00%	4.90	0.00	0.00	0.00
20.00%	5.01	0.11	0.02	0.02
30.00%	5.35	0.34	0.15	0.13
40.00%	5.83	0.48	0.38	0.23
50.00%	6.37	0.54	0.69	0.31

Table 3 Impact on the value of an ITM and OTM call from a change in implied volatility (premiums shown to just two decimal places).

We can make the following conclusions:

- An increase in implied volatility causes the ITM and OTM premiums to increase at an increasing rate.

- At zero implied volatility the premium for the ITM option comprises entirely of intrinsic value. The implied forward price for a spot price of $20.00 (all other things being equal) is $20.20. The option premium returns the present value of the intrinsic value. This is $4.90 [($20.20 - $15.15) / 1.03].
- For the ITM option, intrinsic value is a constant, while for the OTM option consists entirely of time value. As a result table 3 shows how an increase in implied volatility increases the time value component.
- At first glance the OTM option premia appear to be relatively insignificant but note what happens when implied volatility doubles from 20% to 40%. The premium increases from $0.02 to $0.38 – this is an increase of 1,800%. Looked at from this perspective, deeply out of the money options with low implied volatilities may be thought of as cheap lottery tickets. A sudden movement in implied volatility will increase the value of the option by a significant percentage resulting in a massive return on capital.

Change in interest rates

Make sure you reset the spreadsheet to a volatility of 30%. Complete the following table changing only the funding parameter.

Interest rates	Value of call ($)	Change in value of call	Value of put ($)	Change in value of put
1.00%	1.63		1.92	
2.00%	1.69	0.06	1.84	-0.08
3.00%				
4.00%				
5.00%				

Table 4 Impact on the value of a call and a put from a change in the funding rate

Learning points

The main conclusions from table 4 are that an increase in interest rates will increase the value of a call but will reduce the value of a put option. To illustrate why this is so consider how a trader would hedge a short call position. If the option position were to be exercised they will be required to deliver the asset. As such the appropriate hedging strategy is to buy the underlying asset. Although the underlying asset is technically a forward many traders will actually prefer to use the spot market to hedge this potential exposure. The purchase of hedge will require an initial outlay of cash which will need to be financed. An increase in interest rates increases the cost of borrowing and so the option seller will pass this increased charge onto the buyer in the form of a higher premium. The opposite would be true for a put option. The hedge in this case is a short position in the underlying. The money received from the sale of the asset can be put on deposit to earn interest. An increase in interest rates will reduce the cost of carrying the position and hence the premium will be lower.

Change of dividend yield

Make sure you reset the spreadsheet to have a funding rate of 3%. Complete the following table changing only the yield parameter.

Dividend yield	Value of call ($)	Change	Value of put ($)	Change
1.00%	1.84		1.69	
2.00%	1.75	-0.09	1.75	0.06
3.00%				
4.00%				
5.00%				

Table 5 Impact on the value of a call and a put from a change in dividend yields

Learning points

Table 5 shows that an increase in dividend yields decreases the value of the call but increases the value of a put. The seller of a call will be long the underlying asset as a hedge and will therefore receive the dividend yield. An increase in the dividend yield reduces the cost of carrying the hedge and so the benefit is passed onto the buyer in the form of a lower premium.